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Turbulent Boundary-Layer Simulation with an Array of Loudspeakers

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The feasibility is discussed of simulating a random pressure field having the same spatial correlation function as a turbulent boundary-layer pressure field using an array of loudspeakers. This approach could provide a cost-effective laboratory method of measuring the boundary-layer noise transmitted through aircraft fuselage structures. Initially, a theoretical model is used to predict the vibroacoustic response of randomly excited panels. A method of generating a pressure field with predefined statistical properties using an array of loudspeakers is then introduced. Results are obtained in a typical test case for the simulation of boundary-layer-induced noise. It is shown how the number of loudspeakers required to achieve a reasonable approximation of the boundary-layer excitation scales with frequency. It is found that a coarse reproduction of the boundary-layer excitation, using a reduced set of loudspeakers, can still give a good approximation of the panel vibroacoustic response, thus suggesting that direct simulation of the panel response to a boundary-layer excitation using loudspeakers could be feasible.

Nomenclature

A = area of each panel element
c = sound speed
D_P = rigidity of the panel
d = height between the panel and the microphones below the panel
$\boldsymbol{d}$ = $(N_x N_y)$ vector of turbulent boundary-layer (TBL) wall pressures at the microphones positions
$d(M, M')$ = distance between two points M and M'
E_P = Young's modulus of the panel
$E[x]$ = mathematical expectation of a random variable x
$\boldsymbol{e}$ = $(N_x N_y)$ vector of error signals at the microphones outputs
$\boldsymbol{G}$ = $(N_x N_y \times NL_x NL_y)$ plant response matrix between the microphones and the loudspeakers
G_p = panel acoustic response to a unit point force excitation
G_v = panel velocity response to a unit point force excitation
$\boldsymbol{G}_{\{v,p\}}$ = $(N_x N_y \times N_x N_y)$ transfer mobility matrix and the $(PM_x PM_y \times N_x N_y)$ transfer impedance matrix, respectively
h = height between the loudspeakers and the microphones above the panel
h_P = panel thickness
J_e, J_{e_p}, J_{e_v} = normalized mean-square error signals respectively for the approximate TBL field, panel acoustic response, and panel velocity response
k = acoustic wave number
$L_{\{y,x\}}$ = correlation lengths of the TBL along the y and x axes
$l_{\{y,x\}}$ = panel dimensions along the y and x axes
m_r = modal generalized mass term
$N_{\mathrm{col},\{y,x\}}$ = number of correlation lengths to reproduce along the y and x axes
N_L = total number of loudspeakers
$N_{\{y,x\}}$ = number of discrete elements on the panel along the y and x axes
$NL_{\{y,x\}}$ = number of loudspeakers along the y and x axes
$NM_{\{y,x\}}$ = number of microphones above the panel along the y and x axes
P = nondimensional metric for the sound power radiated by the panel
$\boldsymbol{P}$ = $(N_x N_y \times N_x N_y)$ TBL-generating matrix
$PM_{\{y,x\}}$ = number of microphones below the panel along the y and x axes
p_b = "blocked" TBL pressure field
$\wp_\omega^M$ = integral operator to describe the pressure field radiated at point M
R = number of panel modes accounted for in the modal series
$\boldsymbol{R}$ = $(N_x N_y \times N_x N_y)$ radiation resistance matrix
$r_{\{y,x\}}$ = separation distance between two points along the y and x axes
S_{dd} = spectral density function of the TBL excitation
$\boldsymbol{S}_{\mathrm{dd}}$ = $(N_x N_y \times N_x N_y)$ spectral density matrix of the TBL excitation
$\tilde{\boldsymbol{S}}_{\mathrm{ee}}$ = $(N_x N_y \times N_x N_y)$ approximate spectral density matrix of the error signals at the microphones outputs
$\bar{\boldsymbol{S}}_{\mathrm{vv}}$ = spectral density function associated to the plate average velocity
$\tilde{\boldsymbol{S}}_{\mathrm{yy}}$ = $(N_x N_y \times N_x N_y)$ approximate spectral density matrix of the microphones outputs
$S_{\alpha\beta}$ = spectral density functions of the panel vibroacoustic response
$\boldsymbol{S}_{\alpha\beta}$ = spectral density matrices of the panel vibroacoustic response
$\tilde{\boldsymbol{S}}_{\alpha\beta}$ = approximate spectral density matrices of the panel vibroacoustic response
S_Π = spectral density of the sound power radiated by the panel
S_0 = power spectral density of the TBL excitation
Tr = trace of a matrix
U_c = convection velocity of the turbulent flow
V = nondimensional metric for the panel average velocity response
$\boldsymbol{W}$ = $(NL_x NL_y \times N_x N_y)$ control filter matrix
$\boldsymbol{W}_{\mathrm{opt}}$ = $(NL_x NL_y \times N_x N_y)$ optimal matrix of control filter
$\boldsymbol{x}$ = $(N_x N_y)$ vector of white-noise reference signals

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$\mathbf{y}$	=	$(N_x N_y)$ vector of microphones outputs
z	=	height between the microphones above the panel and the panel
α_P	=	Skudrzyk plate constant
$\alpha_{\{y,x\}}$	=	empirical coefficients related to the turbulent flow
δ	=	distance between two adjacent loudspeakers
η	=	damping ratio of the panel
λ	=	acoustic wavelength
μ_P	=	panel density per unit area
ν	=	Poisson ratio of the panel
ρ	=	fluid density
ρ_P	=	mass density of the panel
Σ	=	panel surface
φ_r	=	rth panel structural mode
ω	=	angular frequency
ω_r	=	rth panel natural frequency

Superscripts

H	=	transpose conjugate of a matrix
†	=	pseudoinverse of a matrix

I. Introduction

IN many aeronautical applications, the measurements of the response of structures to turbulent boundary-layer excitation (TBL) is of major importance in order to formulate simplified theoretical models of the problem and to test potential solutions for the reduction of the TBL-induced noise, as generated in the cabin of many commercial aircraft under cruise conditions. In-flight measurements or testing in an anechoic wind tunnel are expensive and time-consuming processes, unsuitable for systematic parametric studies. An alternative approach involves simulating the random pressure field caused by the TBL in the laboratory. Early investigations[1,2] considered the possibility of testing aircraft structures in a reverberant transmission suite in order to measure the response of typical fuselage panels under the assumption that panels excited by a TBL and panels excited by a diffuse acoustic field of equivalent level will respond in the same way. However, it has been shown from recent theoretical work[3] that the two responses can be very different. Differences of 20 dB or so have been observed[4] in certain frequency ranges for the spectral density of the sound power radiated by a typical aircraft panel either excited by a TBL or a diffuse acoustic field, both of unit mean-square pressure level. The differences are caused by the different spatial correlation structures of the two forms of excitation and subsequently the way they couple with the panel structural modes: the panel modes are much more efficiently excited with a diffuse acoustic field than with a TBL field and over a broader frequency range.[4] Therefore, in practice, boundary-layer noise testing in a transmission suite facility is of limited use.

Fahy[5] has examined the problem of simulating TBL pressure fluctuations on suitable scaled models using a siren tube or a single loudspeaker in order to reproduce the decay characteristics of the correlation function of a TBL pressure field. It is argued that at high frequencies these systems will generate acoustic waves, which are correlated over an area much larger than the extent over which the TBL pressure field is correlated. The use of such systems to reproduce boundary-layer noise excitation would be limited over a broad frequency range. The use of an array of shakers or loudspeakers is also considered by this author, but, in 1966, the author concluded that the electrical and mechanical difficulties of implementing such a system would make it a difficult practical proposition. However, it was observed that an approximation to the required pressure distribution could in theory be made[5] with an array of actuators fed by signals recorded from a set of similarly distributed pressure sensors located beneath the actual turbulent boundary layer.

Robert and Sabot[6] have taken another approach: they have addressed the theoretical problem of directly simulating, with an array of shakers driven by random signals, the structural response of a plate excited by a turbulent boundary layer with a small freestream velocity (up to 10 m/s). In this configuration, it can be observed that the extent over which the TBL pressure field is correlated is much smaller than the panel area for frequencies above a fraction of the first natural frequency of the structure. Therefore, the panel structural modes are excited almost independently by the TBL pressure field,[7] and the response of the panel, assumed lightly damped, is dominated by a small number of resonant modes, at least in the low-frequency domain. Simulation results have shown that the response of the TBL-excited panel could be reproduced with an error less than 1 dB up to 1 kHz with only five shakers acting on the panel. However, the method of simulation proposed by Robert and Sabot[6] assumes that the mode shapes are accurately known and some preliminary modal analysis would be required before the drive signals to the shakers could be determined. The TBL simulation could be made completely independent of the properties of the structure when using an array of actuators to reproduce the spatial correlation function of the TBL pressure field. However, it is likely that such a general approach might require a large number of actuators for an accurate simulation of the TBL pressure field, especially at high frequencies.

In the present study, we consider the generation of a random pressure field having the same spatial correlation structure as a TBL pressure field using an array of loudspeakers. The main objective of this paper is to determine the number of loudspeakers required to simulate with an acceptable accuracy the correlation structure of the TBL excitation on a typical aircraft panel up to a certain frequency. It is shown how different geometrical configurations for the system affect the accuracy with which the TBL field is reproduced. A central issue is found to be the error induced on the approximation of the panel vibroacoustic response.

The physical characteristics of the system under study are presented in the next section, together with a discretized model for the vibroacoustic response of the TBL-excited panel. The validation of this model is obtained by comparison with the exact solution on a test case. In the third section, the least-squares approximation to the TBL pressure field with an array of loudspeakers is described, and simulation results obtained in the test case defined in the preceding section are discussed. In particular, the number of loudspeakers, their height above the panel, and the height between the panel and the surface of simulation have been varied.

II. TBL-Excited Panel Model

A. Physical Characteristics of the System

We consider a two-dimensional array of $NL_y \times NL_x$ loudspeakers, evenly spaced in the streamwise and in the spanwise directions, as shown in Fig. 1. The axes y and x correspond, respectively, to the streamwise and to the spanwise directions of the turbulent flow. The loudspeakers array is located at a height h above a set of $NM_y \times NM_x$ evenly spaced microphones that defines the surface of simulation. The grid of microphones is positioned a distance z apart from a finite plate of dimensions $l_y \times l_x$. Unless otherwise stated, the surface of simulation is the panel surface, that is, $z = 0$. The panel

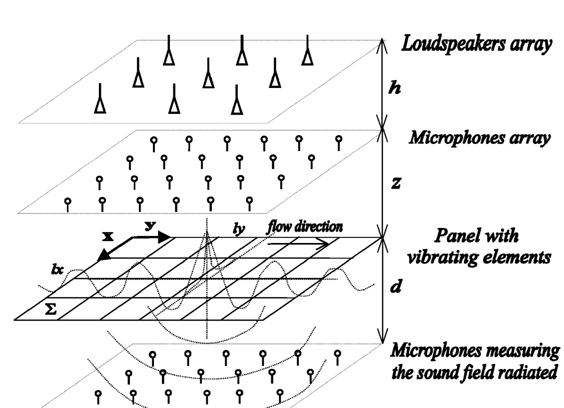

Fig. 1 Use of a two-dimensional array of loudspeakers positioned above a grid of microphones whose outputs are arranged to be as close as possible to the outputs if the microphones were subject to a TBL pressure field excitation.

Table 1 Typical aircraft panel parameters

Parameter	Value
Dimensions	$l_x = 0.328$ m, $l_y = 0.768$ m
Thickness	$h_P = 0.0016$ m
Mass density	$\rho_P = 2700$ kg m^{-3}
Young's modulus	$E_P = 7 \times 10^{10}$ Pa
Poisson ratio	$\nu = 0.33$
Damping ratio	$\eta = 0.02$

is assumed to be set in a rigid baffle. The acoustic pressure field radiated by the plate in the lower half-plane is measured by a plane array of $PM_y \times PM_x$ microphones located at a distance d below the plate. The TBL pressure field that one attempts to reproduce is the "blocked" field, that is, that which would be observed in the absence of panel vibration. Therefore, the loudspeakers are modeled as acoustic sources radiating over a rigid surface.

The physical and geometrical characteristics of the system are now chosen. The correlation structure assumed for the blocked TBL pressure field p_b is that of the Corcos model,[8] so that the cross spectral density of the pressure at two points M and M', which are a distance r_x apart in the spanwise direction and r_y apart in the streamwise direction, is given by

$$S_{\mathrm{dd}}(M - M'; \omega) = \int_{-\infty}^{\infty} \overline{p_b(x, y, t) p_b(x + r_x, y + r_y, t + \tau)} e^{-j\omega\tau} \, \mathrm{d}\tau$$
$$= S_0(\omega) e^{-|r_x|/L_x} e^{-|r_y|/L_y} e^{-j\omega r_y/U_c} \tag{1}$$

where the overbar denotes space–time averaging, $S_0(\omega)$ is the power spectral density at any point, L_y and L_x denote the correlation lengths respectively in the streamwise and in the spanwise directions, and U_c is the convection velocity. The correlation lengths are assumed to be inversely proportional to frequency, as suggested by Corcos, and have the form

$$L_x = U_c/\alpha_x\omega, \qquad L_y = U_c/\alpha_y\omega \tag{2}$$

where α_x and α_y are empirical constants, which are taken to be 0.7 and 0.116 in the following simulations, and the convection velocity is assumed to be 92 ms^{-1}, which corresponds to a freestream velocity of about 115 ms^{-1}. The Corcos model[8] has been obtained from an empirical fit to wall-pressure cross-correlation measurements, and it is simple enough to enable extensive simulations to be performed without a lot of computational effort. Because this model assumes that cross correlations are independent of boundary-layer thickness, it overestimates the correlation lengths at very low frequencies. However, the effect on the results of a more refined model dependent on the boundary-layer thickness is discussed in Sec. III.

We consider a simply supported, untensioned, aluminum panel, whose surface Σ, shown in Fig. 1, is set in a rigid baffle. The panel's parameters are defined in Table 1. The parameters are representative of a skin panel of an aircraft fuselage excited by a turbulent boundary layer under cruise conditions.[9] At this early stage and for sake of simplicity, the influence of membrane stresses as a result of cabin pressurization is not considered in the panel model. This assumption will not affect the results obtained for TBL simulation because the simulation process proposed in this study is independent of the panel properties. However, the error induced on the approximation of the panel vibroacoustic response is dependent on the panel properties and this point will be discussed in Sec. III.

B. Discretized Formulation for the Vibroacoustic Problem

The statistical properties of the panel response to a TBL excitation are first expressed in terms of the statistical properties of the TBL excitation and in terms of the panel response to a point force excitation (or Green functions). The Green functions are then expanded in terms of the panel structural modes, and analytical modal expressions are obtained for the spectral density functions describing the response of the TBL-excited panel. A discretized formulation can be deduced from these expressions by simply approximating the surface integrals using a rectangle quadrature.

First, it can be shown that the spectral density function $S_{\alpha\beta}$ of the panel in response to the spectrum S_{dd} of the TBL excitation is given by[3]

$$S_{\alpha\beta}(M, M'; \omega) = \iint_\Sigma \iint_\Sigma G_\alpha(M, M''; \omega) S_{\mathrm{dd}}(M'' - M'''; \omega)$$
$$\times G^*_\beta(M', M'''; \omega) \, \mathrm{d}\Sigma(M'') \, \mathrm{d}\Sigma(M''') \tag{3}$$

where α and β could denote either the pressure p or the velocity v. $G_p(M, M''; \omega)$ and $G_v(M, M''; \omega)$ represent the contribution, respectively, to the pressure and to the velocity response of the panel at a point M for a harmonic point force excitation acting normally on the panel at the point M''. These quantities are also called panel's Green functions.

The expression for the spectrum S_Π of the sound power radiated by the panel in the lower half-plane reads

$$S_\Pi(\omega) = \frac{1}{2}\Re\left[\iint_\Sigma S_{\mathrm{pv}}(M, M; \omega) \, \mathrm{d}\Sigma(M)\right] \tag{4}$$

where an expression for S_{pv} can be deduced from Eq. (3). Using an integral representation of the pressure field radiated by the panel,[10] it can also be written as

$$S_{\mathrm{pv}}(M; \omega) = j\rho\omega \iint_\Sigma S_{\mathrm{vv}}(M, Q; \omega) \frac{e^{-jkd(M,Q)}}{2\pi d(M, Q)} \, \mathrm{d}\Sigma(Q)$$
$$\equiv \wp^M_\omega(S_{\mathrm{vv}}) \tag{5}$$

where $d(M, Q)$ denotes the distance between two points M and Q, and $k = \omega/c$ is the acoustic wave number. $\wp^M_\omega$ is an integral operator used to calculate at the point M the radiated pressure as a result of a given velocity distribution on the panel.

The panel's Green functions G_v and G_p that appear in Eq. (3) can then be expanded in terms of the panel structural modes as follows[3]:

$$G_v(M, M''; \omega) = j\omega \sum_{r=1}^{R} \frac{\varphi_r(M'')\varphi_r(M)}{m_r\left[(1 + j\eta)\omega_r^2 - \omega^2\right]} \tag{6}$$

$$G_p(M, M''; \omega) = j\omega \sum_{r=1}^{R} \frac{\wp^{M''}_\omega(\varphi_r)\varphi_r(M)}{m_r\left[(1 + j\eta)\omega_r^2 - \omega^2\right]} \tag{7}$$

$$\varphi_r(M) = \varphi_{\{s,t\}}(x, y) = \sin\left(\frac{s\pi x}{l_x}\right)\sin\left(\frac{t\pi y}{l_y}\right)$$

$$\omega_r = \alpha_P^2\left[\left(\frac{s\pi}{l_x}\right)^2 + \left(\frac{t\pi}{l_y}\right)^2\right] \tag{8}$$

where $\alpha_P = (D_P/\mu_P)^{1/4}$ is the Skudrzyk's plate constant that is expressed in terms of the plate rigidity $D_P = E_P h_P^3/12(1 - \nu^2)$ and the panel density per unit area $\mu_P = \rho_P h_P$. m_r is equal to $\mu_P l_x l_y/4$.

After substituting expressions (5–8) into expressions (3) and (4), one obtains the analytical modal formulation related to the spectral quantities of interest.[3] A discretized version of the model is readily obtained if we assume that the panel surface is made up of a number of $N_x \times N_y$ elemental contiguous elements. Each of these elemental surfaces has a piston motion. All of the surface integrals that appear in the aforementioned modal formulation can then be approximated by using a rectangle quadrature formula on the discretized surface, as follows:

$$\iint_\Sigma f(M) \, \mathrm{d}\Sigma(M) \approx A \sum_{n=1}^{N_x N_y} f(M_n) \tag{9}$$

where each elemental area is given by $A = l_x l_y/(N_x N_y)$.

We note that the number of elements $N_x N_y$ used for the calculation of the various integrals can be different according to the nature of the integral (elemental driving, vibrating, or radiating surfaces). In a first approach and for sake of simplicity, we assume that the number of driving pressures, which physically corresponds to the number of microphones on which the TBL pressure field is reproduced, is equal to the number of elemental vibrating surfaces covering the panel and is also equal to the number of elemental radiators. Thus we have $N_x = NM_x$ and $N_y = NM_y$.

From the discretized version of integral (3), the spectral density matrix $\boldsymbol{S}_{\alpha\beta}$ for the vibroacoustic response of the panel can be written in terms of the corresponding plant response matrices $\boldsymbol{G}_\alpha$, $\boldsymbol{G}_\beta$ and in terms of the spectral density matrix $\boldsymbol{S}_{\mathrm{dd}}$ of the TBL excitation in the following condensed form:

$$\boldsymbol{S}_{\alpha\beta} = \boldsymbol{G}_\alpha \boldsymbol{S}_{\mathrm{dd}} \boldsymbol{G}_\beta^H, \qquad \alpha, \beta \in \{v, p\} \tag{10}$$

The entries of these matrices are indicated in the nomenclature. $\boldsymbol{G}_v$ can be interpreted as the matrix of responses between the panel vibration and driving forces acting over the panel (or transfer mobility matrix), and $\boldsymbol{G}_p$ as the matrix of responses between the sound pressure radiated below the panel over a set of $PM_y \times PM_x$ microphones and the panel vibration (or transfer impedance matrix). For sake of consistency, we assume that $PM_x = N_x$ and $PM_y = N_y$.

After discretizing integrals (4) and (5), one obtains the following expression for the spectrum of the sound power radiated by the TBL-excited panel:

$$S_\Pi(\omega) = \frac{\rho\omega A^2}{4\pi}\left\{2\sum_{n=2}^{N_x N_y}\sum_{m=1}^{n-1}\Re[S_{\mathrm{vv}}(M_n, M_m;\omega)]\frac{\sin[kd(M_n, M_m)]}{d(M_n, M_m)} + k\sum_{n=1}^{N_x N_y} S_{\mathrm{vv}}(M_n, M_n;\omega)\right\} \tag{11}$$

where the summation is done over each element of the panel. A concise matrix formulation of expression (11) can be written as follows:

$$S_\Pi = Tr[\boldsymbol{S}_{\mathrm{vv}}\boldsymbol{R}] \tag{12}$$

where $\boldsymbol{R}$ is the radiation resistance matrix for the discretized model.[11]

Expressions (10) and (12) are used to predict the following nondimensional metrics for the panel average velocity response to a TBL excitation and for the subsequent sound power radiated:

$$V = \frac{\omega^2\mu_p^2\bar{S}_{\mathrm{vv}}}{S_0}, \qquad P = \frac{\omega\mu_p S_\Pi}{l_x l_y S_0} \tag{13}$$

where $\bar{S}_{\mathrm{vv}} = Tr[\boldsymbol{S}_{\mathrm{vv}}]/N_x N_y$ is the spectral density associated to the plate average velocity.

In Figs. 2 and 3, calculations based on the discretized approach have been compared to results obtained with an analytical modal formulation[3] for both the metric V and P. A set of 18×8 structural modes were required, respectively, in the streamwise and in the spanwise directions in order to achieve convergence in the vibroacoustic response for the exact solution up to 2 kHz. A grid of $N_y \times N_x = 28 \times 12$ elements has been considered for the discretized solution with the ratio between the number of elements used in the streamwise and the spanwise directions taken equal to the ratio between the panel dimensions in these respective directions ($N_y/N_x = l_y/l_x$).

From Fig. 2, it can be seen that with this number of elements the discretized solution converges up to 800 Hz for the prediction of the metric V. This corresponds to an error lower than 1% for the total average velocity response calculated up to 800 Hz. From Fig. 3, we observe that with the same number of elements both the exact and discretized solutions for the prediction of the metric P agree reasonably well up to 1.2 kHz, with an error lower than 2% in the total sound power radiated up to 1.2 kHz. By comparison with the results obtained in Fig. 2, it seems that the number of elements used to correctly predict the vibration response up to 800 Hz is sufficient to predict the acoustic response up to 1.2 kHz.

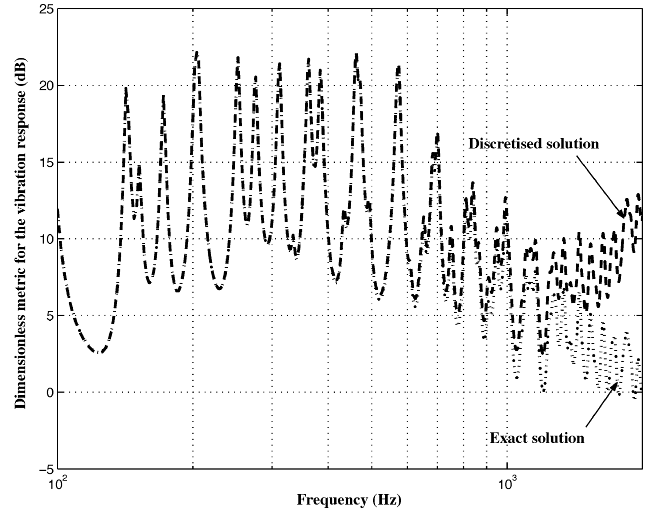

Fig. 2 Narrowband comparison between the exact solution and the discretized solution for the prediction of the nondimensional metric *V* related to the plate average velocity.

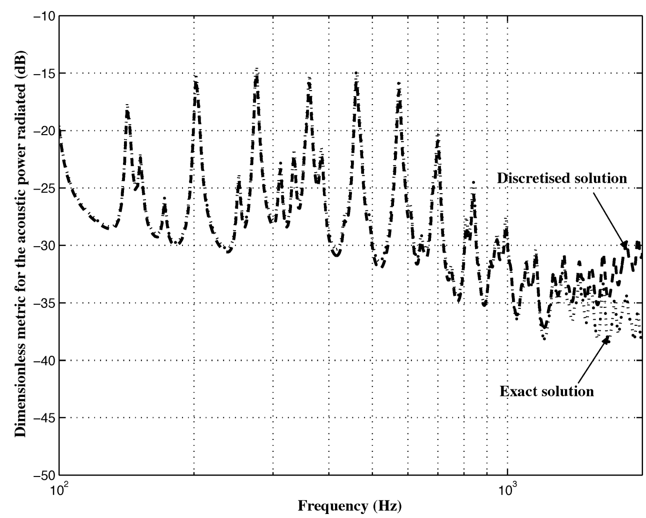

Fig. 3 Narrowband comparison between the semi-analytical solution and the elemental solution for the prediction of the nondimensional metric *P* related to the acoustic power radiated by the plate.

This is because, above 800 Hz, most of the low-order modes are excited above their individual critical frequencies and so are very efficient radiators although they are not resonant.[12] Moreover, most of them are well excited by the TBL excitation field, and these two factors explain why they significantly contribute to the panel acoustic response above 800 Hz with respect to the resonant modes that are weakly excited and very inefficient radiators over this frequency range. Therefore, the number of elements chosen to correctly describe the levels of structural response up to 800 Hz also accounts for the contribution to the panel acoustic response of low-order structural modes radiating efficiently at frequencies well above their resonant frequencies.

Thus, with 28 and 12 elements in the streamwise and spanwise directions the vibroacoustic response of the test panel can be predicted up to 800 Hz with an error lower than 1% and at a reasonable computational cost. This frequency range covers the whole hydrodynamic coincidence frequency range, which extends up to 540 Hz in this configuration. Because this number of elements is assumed to be equal to the number of microphones where the TBL driving pressures are reproduced, it will fix an upper limit for the number of loudspeakers required to simulate the correlation structure of the TBL excitation over the panel surface.

III. TBL Simulation with an Array of Loudspeakers

A series of computer simulations are carried out using the discretized model, in which the panel is assumed to be driven by a set of pressures that reproduce a TBL but are generated by an array of loudspeakers over each elemental surface (see Fig. 1). The induced vibroacoustic response of the panel can then be calculated.

A. Least-Squares Approximations

In this subsection, the least-squares approximation to the TBL pressure field with an array of loudspeakers is presented. The situation is illustrated by the block diagram shown in Fig. 4. First, it is assumed that the pressures at the microphone positions in the TBL wall-pressure fluctuations $\boldsymbol{d}$ are derived from a set of uncorrelated white reference signals $\boldsymbol{x}$ via a matrix of filters $\boldsymbol{P}$ derived from an eigenanalysis of $\boldsymbol{S}_{\mathrm{dd}}$ (Ref. 13). The reference signals are also used to drive a matrix of control filters $\boldsymbol{W}$ that determines the input signals to an array of loudspeakers, which in turn drive the microphone outputs $\boldsymbol{y}$ via the matrix of plant response $\boldsymbol{G}$ between the microphones and the loudspeakers. The problem is then how to best design the matrix of control filters to ensure that the statistical properties of the microphone outputs $\boldsymbol{y}$ are most similar to those from the TBL-excited panel $\boldsymbol{d}$ for a given arrangement of loudspeakers and microphones.[14,15] As shown in Fig. 4, the vector of error signals at the microphones outputs and at a given frequency $\boldsymbol{e}$ is defined to be

$$\boldsymbol{e} = (\boldsymbol{P} - \boldsymbol{G}\boldsymbol{W})\boldsymbol{x} \tag{14}$$

The cost function being minimized is the sum of the mean-square error signals normalized by the sum of the mean-square microphones signals as a result of the TBL. If the cost function is small enough, then the spectral density matrix of the microphone outputs $\tilde{\boldsymbol{S}}_{\mathrm{yy}} = E[\boldsymbol{y}\boldsymbol{y}^H]$ must match the spectral density matrix generated at the microphones by a turbulent boundary layer $\boldsymbol{S}_{\mathrm{dd}} = E[\boldsymbol{d}\boldsymbol{d}^H]$. This is achieved for the optimum least-squares matrix of filters given by[15,16]

$$\boldsymbol{W}_{\mathrm{opt}} = [\boldsymbol{G}^H\boldsymbol{G}]^{-1}\boldsymbol{G}^H\boldsymbol{P} = \boldsymbol{G}^{\dagger}\boldsymbol{P} \tag{15}$$

where $\boldsymbol{G}^{\dagger}$ is the pseudoinverse of $\boldsymbol{G}$. The minimum normalized mean-square error can be shown to be[17]

$$J_e(\boldsymbol{W}_{\mathrm{opt}}) = \frac{Tr\left[(\boldsymbol{I} - \boldsymbol{G}\boldsymbol{G}^{\dagger})\boldsymbol{S}_{\mathrm{dd}}\right]}{Tr[\boldsymbol{S}_{\mathrm{dd}}]} \tag{16}$$

The normalized mean-square error signals induced on the structural and acoustic response of the panel when subject to the approximate TBL pressure field generated using an array of loudspeakers are, respectively, given by

$$J_{e_v}(\boldsymbol{W}_{\mathrm{opt}}) = \frac{Tr\left[\boldsymbol{G}_v\tilde{\boldsymbol{S}}_{\mathrm{ee}}(\boldsymbol{W}_{\mathrm{opt}})\boldsymbol{G}_v^H\right]}{Tr\left[\boldsymbol{G}_v\boldsymbol{S}_{\mathrm{dd}}\boldsymbol{G}_v^H\right]} \tag{17}$$

$$J_{e_p}(\boldsymbol{W}_{\mathrm{opt}}) = \frac{Tr\left[\boldsymbol{G}_p\boldsymbol{G}_v\tilde{\boldsymbol{S}}_{\mathrm{ee}}(\boldsymbol{W}_{\mathrm{opt}})(\boldsymbol{G}_p\boldsymbol{G}_v)^H\right]}{Tr\left[\boldsymbol{G}_p\boldsymbol{G}_v\boldsymbol{S}_{\mathrm{dd}}(\boldsymbol{G}_p\boldsymbol{G}_v)^H\right]} \tag{18}$$

where $\tilde{\boldsymbol{S}}_{\mathrm{ee}}(\boldsymbol{W}_{\mathrm{opt}}) = [\boldsymbol{G}\boldsymbol{G}^{\dagger} - \boldsymbol{I}]\boldsymbol{S}_{\mathrm{dd}}[(\boldsymbol{G}\boldsymbol{G}^{\dagger})^H - \boldsymbol{I}]$. The approximate spectral density matrix for the TBL pressure field is given by

$$\tilde{\boldsymbol{S}}_{\mathrm{yy}} = \boldsymbol{G}\boldsymbol{G}^{\dagger}\boldsymbol{S}_{\mathrm{dd}}(\boldsymbol{G}\boldsymbol{G}^{\dagger})^{\boldsymbol{H}} \tag{19}$$

The corresponding spectral density matrices for the structural and acoustic responses of the panel excited by the approximate TBL field take the form

$$\tilde{\boldsymbol{S}}_{\alpha\alpha} = \boldsymbol{G}_{\alpha}\tilde{\boldsymbol{S}}_{\mathrm{yy}}\boldsymbol{G}_{\alpha}^H, \qquad \alpha \in \{v, p\} \tag{20}$$

B. Simulation Results

A first series of computer simulations has been performed in which the loudspeakers have been modeled as acoustic monopoles radiating over a rigid surface, and their number $NL_y \times NL_x$ and height h above the panel surface have been varied. The ratio between the number of loudspeakers as well as the number of microphones along each direction of the panel is constant and is chosen equal to the ratio between the panel dimensions along the respective directions, that is, $NL_y/NL_x = l_y/l_x \approx 2.3$. In the test case of interest, this choice corresponds to a two-dimensional array of loudspeakers that are fairly uniformly distributed. This uniform arrangement of loudspeakers might not be the most efficient when the correlation lengths are different in the two directions, but provides a convenient starting point.

Unless otherwise stated, simulations have been performed in order to reproduce 1.4×5.2 correlation lengths over an array of 28×12 evenly spaced microphones respectively in the streamwise and in the spanwise directions, which corresponds to a frequency of $f = 275$ Hz from Eq. (2). This choice falls within the frequency range of convergence of the discretized solution. Indeed, with reference to the results presented in the preceding section, we have shown that if the spectral density matrix of the TBL pressure field is given on a grid of 28×12 elements, then the discretized solution allows to predict with confidence the vibroacoustic response of the panel up to 800 Hz, that is, over an area that includes up to 4.2 correlation lengths in the streamwise direction and up to 15 correlation lengths in the spanwise direction. Note that the number of correlation lengths $N_{\mathrm{col},y}$ and $N_{\mathrm{col},x}$ that we aim to reproduce along each direction of the panel satisfies the relation $N_{\mathrm{col},y}/N_{\mathrm{col},x} = l_y\alpha_y/(l_x\alpha_x) \approx 0.3$.

Figure 5 shows the variation of the normalized mean-square error in the pressure on the panel, as given by Eq. (16), as a function of the height of the loudspeaker array normalized by the distance between two adjacent loudspeakers $\delta = l_x/NL_x = l_y/NL_y$ for different numbers of loudspeakers in the array. As observed in the previous study of the one-dimensional case,[17] very little reduction in the mean-square error is achieved when we account for a coarse grid of loudspeakers and for heights small compared with the separation distance δ. However, better reductions are achieved with a denser array of loudspeakers, and these reductions are almost independent of the height of the loudspeaker array provided it is greater than the loudspeaker separation δ. If the loudspeakers are too far from the microphones, there might be numerical conditioning problems, however, and a reasonable height h in order to obtain a maximum error reduction at this frequency appears to be equal to the separation distance δ. In the parametric studies that will be subsequently presented, the loudspeakers are positioned as just described, that is, as far from the microphone array as the distance they are apart from each other.

Figure 6 shows the reduction in the normalized mean-square error, as given by Eqs. (16–18), and associated with the approximate TBL

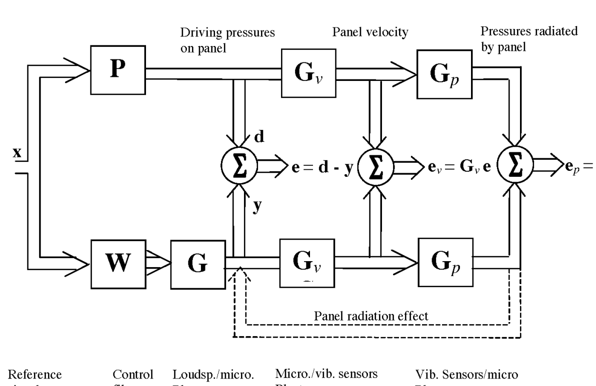

Fig. 4 Block diagram for the calculation of the least-squares control filters to reproduce the TBL pressures $\boldsymbol{d}$ over the panel surface ($z = 0$) and influence on the error signals e_v and e_p, respectively, related to the vibrating and acoustic responses of the panel, derived from the discretized model.

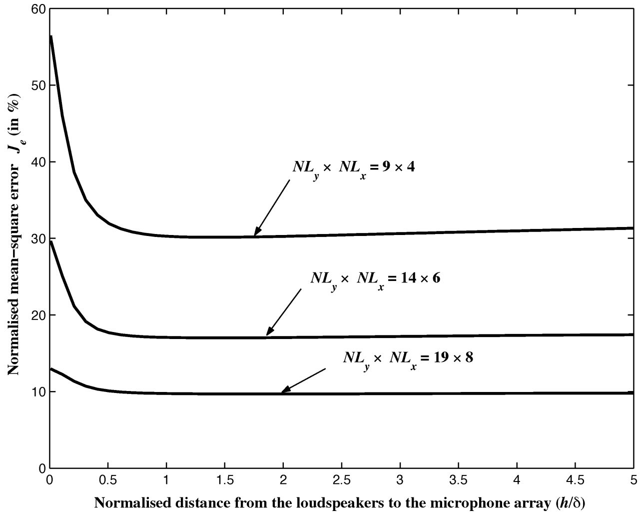

Fig. 5 Normalized mean-square error J_e for the simulations in which various numbers of loudspeakers $NL_y \times NL_x$, were used to reproduce the TBL pressure field at 275 Hz for which there are 1.4 × 5.2 correlation lengths, respectively, in the streamwise and in the spanwise directions, as a function of the normalized distance of the loudspeakers grid to an array of 28 × 12 microphones positioned over a rigid panel.

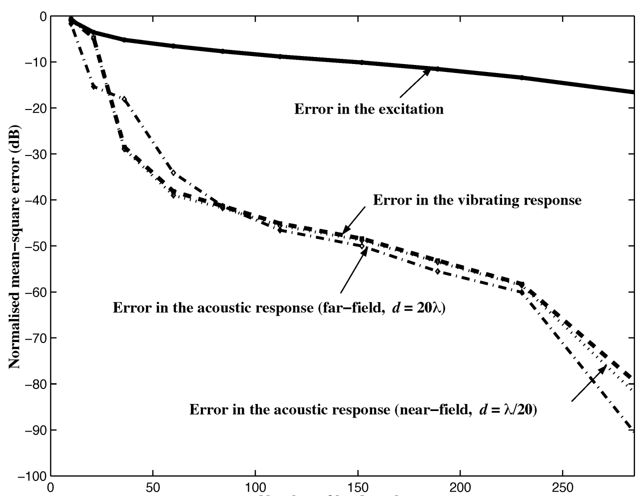

Fig. 6 Normalized mean-square error J_e associated with the approximate TBL pressure field generated using an array of loudspeakers when we aim to reproduce 1.4 × 5.2 correlation lengths respectively in the streamwise and in the spanwise directions over an array of 28 × 12 microphones that is J_{e_v}, associated with the vibrating response induced by the approximate TBL field and calculated over a grid of 28 × 12 elements, and, that is J_{e_p}, of the acoustic response induced by the approximate vibrating response and calculated over a grid of 28 × 12 microphones located in the lower half-plane at various distances from the panel as a function of the number of loudspeakers.

pressure field, with the structural response and with the acoustic response induced by the approximate TBL pressure field driving the panel, as a function of the number of loudspeakers used to simulate the TBL. It can be seen that, for a fixed number of loudspeakers, a much larger reduction is observed in the normalized mean-square error associated with the approximate vibrating and acoustic response than that associated with the approximate TBL pressure field. It can also be seen that the variation with the number of loudspeakers of the mean-square error reduction associated with the acoustic response in the near field ($d = \lambda/20$) closely follows the error reduction related to the panel vibrating response. A larger mean-square error reduction is globally achieved in the far-field acoustic response ($d = 20\lambda$) with respect to the near-field acoustic response, but not uniformly. The main conclusion from these simulations is that, in the aircraft

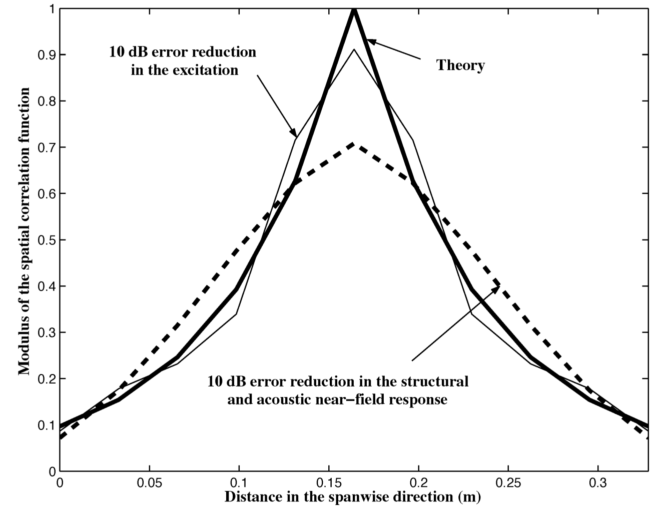

Fig. 7 Modulus of the assumed spatial correlation function of the TBL pressure field in the spanwise direction (——), that of the approximate pressure field generated using 19 × 8 loudspeakers and corresponding to a 10-dB reduction on the excitation error (——), and that of the approximate pressure field generated using 7 × 3 loudspeakers and corresponding to a 10-dB reduction on the error induced on the panel structural and acoustic near-field response (– – –).

panel configuration, a very reduced set of loudspeakers is sufficient to achieve important reductions in the mean-square error associated with the panel vibroacoustic response, whereas a larger number of acoustic sources is required in order to achieve the same level of reduction in the error for the reproduction of the TBL pressure field. This behavior is associated with the wave-number filtering effect of the vibration and acoustic response of the panel on the TBL excitation, so that the panel velocity field and the sound pressure field are much more spatially coherent than the TBL pressure field, and so a smaller number of independent acoustic sources is sufficient for their reproduction. As a consequence, the panel vibroacoustic response is less sensitive than the TBL pressure field to imperfections in the simulation technique. This conclusion clearly depends on the panel properties. Indeed, if the panel were tensioned its modal density within the frequency range of interest would be reduced, and so the TBL excitation spectrum would be even more filtered by the panel resonances. As a result, it is anticipated that even fewer loudspeakers would be required to simulate the vibroacoustic response of a tensioned panel.

Figure 7 shows the spatial variation, in the spanwise direction, of the TBL correlation function as a result of the original Corcos model [Eq. (1)] and the one as a result of the least-squares approximation to the TBL pressure field [Eq. (19)] either when using an array of 19×8 loudspeakers, which corresponds to a 10-dB error reduction in the excitation, or when using an array of 7×3 loudspeakers, which corresponds to a 10-dB reduction in the error induced in the structural and acoustic near-field response. It is observed that even though the peak value of the correlation function when reproduced with 7×3 loudspeakers is about 30% too small the approximate vibroacoustic response of the panel is already very well reproduced. The accuracy in the spatial reproduction of the corresponding correlation structures is now further examined by plotting the spatial correlation structure of the approximate spectral density matrices given by Eqs. (19) and (20) for the TBL driving pressures acting on the panel (Fig. 8; top row), for the velocity response induced on the panel (Fig. 8; middle row) and for the acoustic response induced in the near-field area of the panel (Fig. 8; bottom row). Each figure presents simulation results (right-hand column) performed with an array of only 9×4 loudspeakers together with the assumed correlation structures generated by the TBL (left-hand column). Differences are observed between the correlation structures of the assumed and the approximate TBL pressure field. However, an excellent approximation to the required correlation structure associated with the panel velocity and acoustical response is already achieved.

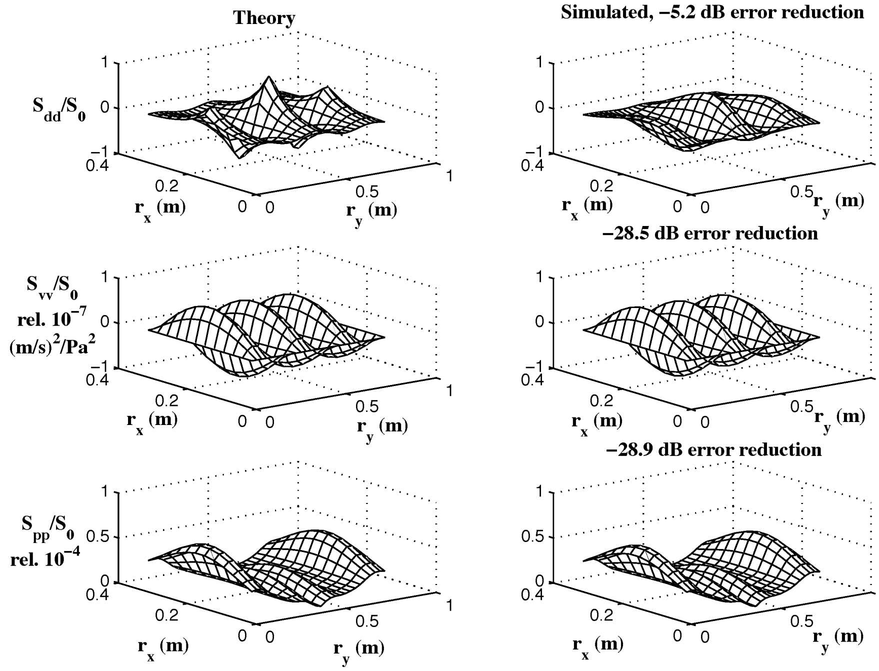

Fig. 8 Spatial correlation structure obtained when perfect reproduction of the TBL pressure field over 1.4 × 5.2 correlation lengths at the microphones locations on the panel is assumed (left-hand column) and that of the approximate generated using 9 × 4 loudspeakers (right-hand column), respectively, for the TBL pressure field (top row), for the panel velocity response (middle row), and for the near-field panel acoustic response (bottom row).

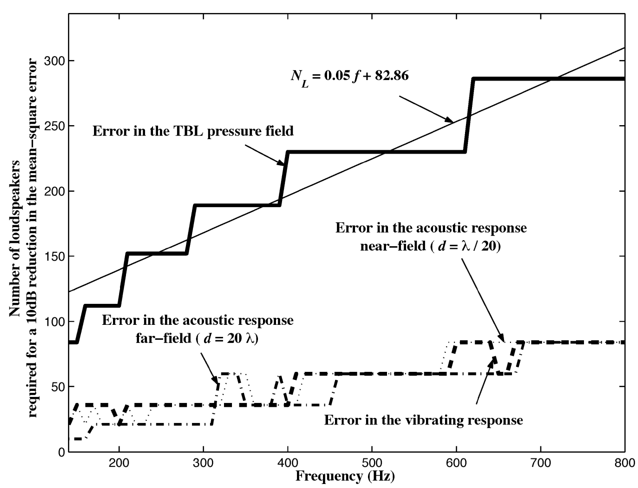

Fig. 9 Number of loudspeakers required for a 10-dB reduction in the mean-square error associated to the TBL pressure field (——), to the panel velocity response (– – –), to the panel acoustic response in the far field (– · –), and to the panel acoustic response in the near field (· · · ·), plotted as a function of frequency.

From the preceding results, we can reasonably assume that the TBL pressure field and the panel vibroacoustic response are sufficiently well reproduced if the mean-square error is reduced by 10 dB. We now investigate the number of acoustic sources required to achieve a 10-dB reduction in the mean-square error associated to the TBL excitation and to the panel response, as a function of frequency, and hence the number of correlation lengths over the panel. Figure 9 shows the number of loudspeakers required for a 10-dB reduction in the mean-square error associated to the TBL pressure field, to the panel vibrating response and to the panel acoustic response either in the near field or in the far field, plotted as a function of frequency. Because the ratio between the number of loudspeakers along each direction of the panel is constant, discrete values for the total number of loudspeakers required for such a reduction are observed. A linear polynomial fitting of the incremental results predicts that the total number of loudspeakers required for the TBL simulation changes with frequency as

$$N_L \approx 0.05f + 82.86 \tag{21}$$

From Fig. 9, it can also be seen that, although 26 × 11 loudspeakers are required for a 10-dB reduction in the mean-square error associated to the TBL reproduction up to 800 Hz, a significantly smaller number of loudspeakers (14 × 6) is sufficient to achieve, over the same frequency range, a similar reduction in the mean-square error induced on the panel vibroacoustic response by the approximation of the TBL excitation.

This result remains almost unchanged when we consider a modified Corcos model[18] taking into account the dependence of correlation lengths on boundary-layer thickness. Indeed, it has been found that, for a 10-dB reduction in the excitation error, a greater number of loudspeakers is required with respect to the original Corcos model [about twice the number predicted by Eq. (21) at frequencies below about 200 Hz], whereas about the same number of loudspeakers (14 × 6) is still sufficient to achieve a 10-dB reduction in the error for the panel response up to 800 Hz.

In practice, the surface of simulation cannot exactly coincide with the panel surface caused by the finite size of the microphones. Further simulations have shown that an accurate reproduction of the TBL pressure field on the panel requires that the height between the grid of microphones and the panel falls below 20% of the separation distance between two adjacent loudspeakers. However, this height only falls below the separation distance between two adjacent loudspeakers for an accurate simulation of the panel vibroacoustic response.

IV. Conclusions

A signal processing formulation has been developed in order to investigate the feasibility of simulating a turbulent boundary-layer (TBL) pressure field over a panel using a two-dimensional array of loudspeakers driven by partially correlated noise sources. Simulations have been performed that correspond to a typical untensioned aircraft panel. The number of loudspeakers and microphones has been chosen in relation to the convergence properties of a discretized solution that predicts the vibroacoustic response of the TBL-excited panel. It has been found that a choice of 28 × 12 elements over the panel correctly describes the vibroacoustic response of the test panel up to 800 Hz and at a reasonable computational cost.

Simulation results obtained in this configuration have shown that a reasonable height between the loudspeakers and the microphones to obtain the maximum mean-square error reduction in the TBL reproduction appears to be the separation distance between two adjacent loudspeakers. The number of loudspeakers required to achieve a 10-dB reduction error for the TBL simulation scales with the frequency. Furthermore, simulations performed up to 4 and 15 correlation lengths respectively along the streamwise and the spanwise directions have shown that a very reduced set of loudspeakers is sufficient to achieve a good approximation of the panel vibroacoustic response, whereas a much larger number of loudspeakers is required in order to achieve the same accuracy for the reproduction of the TBL pressure field. This conclusion remains unchanged if the dependence of TBL spatial correlations on boundary-layer thickness is taken into account. Finally, it has been observed that there is still an accurate reproduction of the TBL excitation on the panel on the source side provided that the height between the grid of microphones and the panel falls below 20% of the separation distance between two adjacent loudspeakers.

Although many practical issues still need to be addressed, the preliminary results presented in this study indicate that there is no fundamental reason why the laboratory simulation of TBL pressure field with an array of loudspeakers could not be achieved. Finally, this system also allows the synthesis of other random fields such as the acoustic diffuse field. This could be applied in the low-frequency domain in order to compensate for the resonant behavior of reverberant suites.[16]

Acknowledgments

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